

# The Jpp - JQuadrature package

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2023-01-11 09:17:03

## Abstract

The Jpp - JQuadrature package provides a set of  $C^{++}$  methods for the numerical computation of specific integrals.

## 1 Introduction

This note describes the  $C^{++}$  methods that are part of the Jpp framework for the numerical computation of specific integrals.

## 2 Numerical integration

In general, the integration of a function can be computed numerically by a weighed sum of the function values at some predetermined abscissas, i.e:

$$\int_{x_1}^{x_2} W(x) f(x) dx \simeq \sum_{i=0}^{N-1} w_i f(x_i) \quad (1)$$

where  $x_1$  ( $x_2$ ) refers to the lower (upper) limit of the integral and  $W(x)$  to some weight function. The summation is performed at fixed values,  $x_i$ , with weights,  $w_i$ . The goal is then to find the values of  $x_i$  and  $w_i$  that yield the most accurate result for a limited number of points,  $N$ . There is extensive literature on the determination of the optimal values for the abscissas and weights [1]. In the following, the classes that are available within the Jpp - JTools package for the numerical computation of function integrals are briefly described.

The base class for the various implementations is a collection of elements, based on the JCollection and JElement2D classes that are part of the Jpp - JTools framework.

```
class JQuadrature :  
    public JCollection<JElement2D_t>  
{}
```

The following classes extend the JCollection class. The methods `getX()` and `getY()` of the JElement2D class correspond to the abscissas and weights, respectively.

## 3 Implementations

### 3.1 Gauss-Legendre integration

The `JGaussLegendre` class can be used to evaluate the abscissas and weights for  $W(x) = 1$ .

```
class JGaussLegendre :
    public JQuadrature
{
    JGaussLegendre(const int n, const double eps);
};
```

The first argument of the constructor refers to the number of points and the second (optional) argument to the precision of the evaluation of the abscissa values.

### 3.2 Gauss-Laguerre integration

The `JGaussLaguerre` class can be used to evaluate the abscissas and weights for  $W(x) = x^\alpha e^{-x}$ .

```
class JGaussLaguerre :
    JQuadrature
{
    JGaussLaguerre(const int n, const double alf, const double eps);
};
```

The first argument of the constructor refers to the number of points, the second argument to the power,  $\alpha$ , and the third (optional) argument to the precision of the evaluation of the abscissa values.

### 3.3 Gauss-Hermite integration

The `JGaussHermite` class can be used to evaluate the abscissas and weights for  $W(x) = e^{-x^2}$ .

```
class JGaussHermite :
    public JQuadrature
{
    JGaussHermite(const int n, const double eps);
};
```

The first argument of the constructor refers to the number of points and the second (optional) argument to the precision of the evaluation of the abscissa values.

## 4 Example

As an example, the effect of the transition-time spread (TTS) of the photo-multiplier tube (PMT) on the probability density function (PDF) of the arrival time of light can approximately be determined by the numerical convolution of the PDF and a Gaussian function with a  $\sigma = \text{TTS}$ .

```
JGaussHermite engine(25);

double x = ..;
double y = 0.0;

for (JGaussHermite::const_iterator j = engine.begin(); j != engine.end(); ++j) {

    const double u = j->getX();
    const double w = j->getY() / sqrt(PI);

    y += w * f(x + u*TTS);
}
```

where  $x$  refers to some arrival time and  $f()$  to the PDF as a function of the arrival time. The result is stored in  $y$ .

## References

- [1] Numerical Recipes in C++, W.H. Press, S.A. Teukolsky, W.T. Vetterling and B.P. Flannery, Cambridge University Press.